EJERCICIO 1 (minuto 8:03)

Calcular los valores de las funciones hiperbólicas de η, de la transformación de Lorentz, fuera del cono, para las que el tiempo t´ es cero

$$X' = \Lambda X$$

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

Buscamos los valores de las funciones que hace t´ igual a cero.

$$\begin{pmatrix} 0 \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ x' \end{pmatrix} = \begin{pmatrix} t\cosh\eta + x\sinh\eta \\ t\sinh\eta + x\cosh\eta \end{pmatrix} = \begin{pmatrix} t & x \\ x & t \end{pmatrix} \begin{pmatrix} \cosh\eta \\ \sinh\eta \end{pmatrix}$$

Calculamos la inversa de la matriz:

$$\begin{pmatrix} t & x \\ x & t \end{pmatrix}^{-1} = \frac{1}{t^2 - x^2} \begin{pmatrix} t & -x \\ -x & t \end{pmatrix}$$

$$\begin{pmatrix} \cosh\eta \\ \sinh\eta \end{pmatrix} = \frac{1}{t^2 - x^2} \begin{pmatrix} t & -x \\ -x & t \end{pmatrix} \begin{pmatrix} 0 \\ x' \end{pmatrix}$$

$$\boxed{\boldsymbol{\cosh\eta = \frac{-x\,x'}{t^2 - x^2}}}$$

$$\boxed{\boldsymbol{\sinh\eta = \frac{t\,x'}{t^2 - x^2}}}$$

$$\boxed{\boldsymbol{\tanh\eta = -\frac{t}{x}}}$$

<u>EJERCICIO 2</u> (minuto 14:06)

Calcular el valor de η, de la transformación de Lorentz, <u>dentro</u> del cono, para que la ordenada x´ sea cero

$$\begin{pmatrix} t´ \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$\begin{pmatrix} \cosh\eta \\ \sinh\eta \end{pmatrix} = \frac{1}{t^2 - x^2} \begin{pmatrix} t & -x \\ -x & t \end{pmatrix} \begin{pmatrix} t´ \\ 0 \end{pmatrix}$$

$$\cosh\eta = \frac{t\, t´}{t^2 - x^2}$$

$$\sinh\eta = \frac{-t´\, x}{t^2 - x^2}$$

$$\tanh\eta = -\frac{x}{t}$$

Siendo:

$$\tanh^{-1} x = \frac{1}{2} \ln\frac{1+x}{1-x}$$

Resulta:

$$\eta = \tanh^{-1}\left(-\frac{x}{t}\right) = \frac{1}{2} \ln\frac{1 + \left(-\frac{x}{t}\right)}{1 - \left(-\frac{x}{t}\right)} = \frac{1}{2} \ln\frac{t-x}{t+x} = \frac{1}{2} \ln\left(\frac{t-x}{t+x}\frac{t+x}{t+x}\right)$$

$$\boxed{\boldsymbol{\eta = \frac{1}{2} \ln\frac{t^2 - x^2}{(t+x)^2}}}$$

<u>EJERCICIO 3</u> (minuto 35:05)

Demostrar que, si

$\boldsymbol{A^{\dagger} = A \ y \ B^{\dagger} = B}$

Entonces:

$$\boldsymbol{[A\,,B]^{\dagger} = -[A\,,B]}$$

$[A\,,B] = AB - BA$

$[A\,,B]^{\dagger} = (AB - BA)^{\dagger} = (AB)^{\dagger} - (BA)^{\dagger} = B^{\dagger}A^{\dagger} - A^{\dagger}B^{\dagger}$

Pero como

$A^{\dagger} = A \ y \ B^{\dagger} = B$

$[A\,,B]^{\dagger} = BA - AB = -(AB - BA)$

$$\boxed{\boldsymbol{[A\,,B]^{\dagger} = [A\,,B]}}$$